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18AI56

Fifth Semester B.E. Degree Examination, June/July 2023
Mathematics for Machine Learning

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define the linear dependent and linear independent of the vector space V(F). Also show that the set of vectors (1 0 1), (1 1 0) (-1, 0, -1) is linearly dependent in V_3(IR). **(06 Marks)**

b. Solve the system of equations and also show that the solution is unique.
$x_1 + x_2 + x_3 = 3$
$x_1 - x_2 + 2x_3 = 2$
$2x_1 + 3x_3 = 1.$ **(06 Marks)**

c. For the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$. Determine the linear transformation
T : V_3(IR) → V_2(IR) relative to the basis B_1 and B_2 of V_3(IR) are V_2(IR).
i) B_1 = {(1 1 1) (1 2 3) (1 0 0)}
ii) B_2 = {(1, 1) (1, -1)} **(08 Marks)**

OR

2 a. Define:
i) An inner product space
ii) Projection of two vectors u and v
iii) Orthogonal vectors
iv) An orthogonal set. **(08 Marks)**

b. Solve by using the Gaussian elimination method
$2x_1 + x_2 + 4x_3 = 12$
$4x_1 + 11x_2 - x_3 = 33$
$8x_1 - 3x_2 + 2x_3 = 20$ **(06 Marks)**

c. Obtain the matrix of linear transformation T : V_2(IR) → V_3(IR), defined by T(x, y) = (x + y, x, 3x − y) with respect to the basis B_1 and B_2 where B_1 = {(1, 1), (3, 1)} and B_2 = {(1, 1, 1), (1, 1, 0), (1, 0, 0)}. **(06 Marks)**

Module-2

3 a. Show that the given vector form an orthogonal basis for R^3 also express $\vec{0}$ as a linear combination of the basis vector, write the coordinate vector $[W]_B$ of $\vec{W}$ with respect to the basis $B = \{\vec{V}_1, \vec{V}_2, \vec{V}_3\}$ of R^3 where $V_1 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$ $V_2 = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$ $V_3 = \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}$ $W = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. **(08 Marks)**

b. Reduce the matrix to diagonal form

$$A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$$ **(06 Marks)**

c. If $v = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ then find the orthogonal projection of v on to u and the orthogonal set. **(06 Marks)**

OR

4 a. Find the singular value decomposition [SVD] of the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. **(10 Marks)**

b. Show that the Eigen values of the following matrix are all equal, and also find the corresponding eigen vector.

$$A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$$ **(10 Marks)**

Module-3

5 a. A particle moves along the curve $\vec{r} = t^2\,\hat{i} - t^3\,\hat{j} + t^4\,\hat{k}$, where 't' is the time. Find the magnitude of tangential component of its acceleration $t = 1$. **(06 Marks)**

b. If $U = x + y + z$, $V = x^2 + y^2 + z^2$, $W = xy + yz + zx$, then prove that grad u, grad v, grad w are coplanar. **(06 Marks)**

c. If $f(x) = \sqrt{x^2 + \exp(x^2)} + \cos(x^2 + \exp(x^2))$ find $\frac{df}{dx}$. Using the following computation graph and the intermediate variables a, b, c, d where $a = x^2$, $b = \exp a$, $c = a + b$, $d = \sqrt{c}$, $e = \cos c$, $f = d + e$.

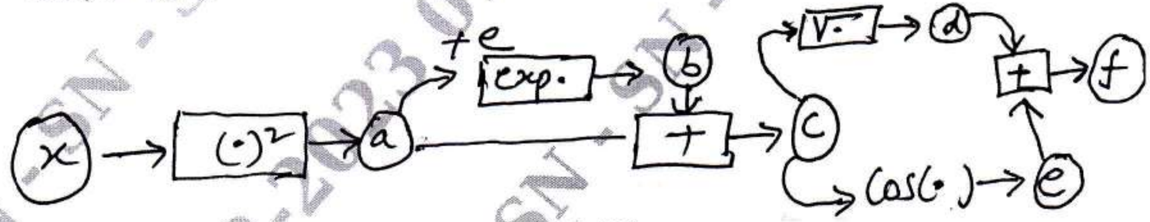

Fig.Q.5(c)

(08 Marks)

OR

6 a. If the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at (-1, 1, 2) has a maximum magnitude of 32 units in the direction of parallel to y-axis find a, b, c. **(08 Marks)**

b. Define gradient of a vector valued function consider the function $h : R \to R$ $h(t) = (f \circ g)t$ $f : R^2 \to R$ and $g : R \to R^2$, if $f(x) = \exp(x_1 x_2^2)$ $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = g(t) = \begin{bmatrix} t\cos t \\ t\sin t \end{bmatrix}$ then compute gradient of h with respect to t. **(12 Marks)**

Module-4

7 a. State and prove Baye's theorem on conditional probability. **(08 Marks)**

b. Let A and B be two events, which are not mutually exclusive and are connected with random experiment. Given that $P(A) = 3/4$, $P(B) = 1/5$, $P(A \cap B) = 1/20$ then find: i) $P(A \cup B)$ ii) $P(A \cap \overline{B})$ iii) $P(\overline{A} \cap B)$ iv) $P(A/B)$ and $P(B/A)$. **(06 Marks)**

c. A random variable x has the following probability distribution:

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Find : i) Value of K ii) $P(x < 6)$ iii) $P(x \geq 6)$. **(06 Marks)**

OR

8 a. Test whether the following function is a density function $f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ if so determine the probability that the variate having its density function will fall in the interval (1, 2). **(08 Marks)**

b. The length of the telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth i) Ends in less than 5 minutes ii) Between 5 and 10 minutes. **(06 Marks)**

c. Define binomial distribution and find the binomial probability distribution which has mean 2 and variance 4/3. **(06 Marks)**

Module-5

9 a. By using gradient descent method (steepest method) for $f(x_1\ x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2$ has the optimal solution starting from the point (0, 0) carry out four iterations. **(12 Marks)**

b. Use Lagrange's multiplier, find the dimension of the rectangular box, which is open at the top of maximum capacity whose volume is 32 cubic feet. **(08 Marks)**

OR

10 a. Given that $x + y + z = a$ where 'a' is a constant, find the extreme value of the function $f(x, y, z) = x^m y^n z^p$. **(08 Marks)**

b. Define a convex and a concave function, test the nature of definiteness by checking its extreme values for the function
$f(x_1\ x_2) = x_1^3 + x_2^3 + 2_1^2 + 4x_2^2 + 6$. **(06 Marks)**

c. Determine whether the function $f(x) = x \log_2 x$ is convex or not for $x > 0$. **(06 Marks)**

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